Int. J. Solids Structures Vol. 30, No. 18, pp. 2485–2499, 1993
Printed in Great Britain

0020–7683/93 $6.00+ .00
Pergamon Press Ltd

THE EFFECT OF LOOSE BOUNDARIES ON WAVE PROPAGATION IN A POROUS SOLID: REFLECTION AND REFRACTION OF SEISMIC WAVES ACROSS A PLANE INTERFACE

A. K. VASHISTH and M. L. GOGNA
Department of Mathematics, Kurukshetra University, Kurukshetra-132 119 (Haryana), India

(*Received* 4 *August* 1992; *in revised form* 9 *March* 1993)

Abstract—Assuming that the interface of two loosely bonded half-spaces, an elastic solid half-space and a liquid-saturated porous solid half-space, permits a finite amount of slip, the problem of reflection and refraction of plane seismic waves incident on such an interface is studied. It is further assumed that stresses are continuous and a linear relation exists between the shearing stress and the slip across the interface. Numerical results are exhibited in graphical form for different degrees of bonding. Two limiting cases of smooth interface and bonded interface are shown to be special cases of this general problem. It is observed that there is an attenuation of energy at a loosely bonded interface which agrees fairly well with the results of the earlier studies.

INTRODUCTION

Biot (1956a, b, 1962a, b) studied the propagation of elastic waves in a fluid-filled porous solid and showed that there are two dilatational waves, propagating with different velocities, and one shear wave. Deresiewicz (1960), Deresiewicz and Rice (1962, 1964) and Deresiewicz and Levy (1967), Geertsma and Smit (1961), Hajra and Mukhopadhyay (1982) discussed the effect of boundaries on wave propagation in a liquid-saturated porous solid. Deresiewicz and Skalak (1963) investigated the appropriate boundary conditions for the boundaries of porous solids.

In the problems of calculating the reflection and transmission coefficients of elastic waves at the interface between two half-spaces, it is usually assumed that the half-spaces are in welded contact which is a reasonable assumption for most situations. In certain situations, however, there are reasons for expecting that bonding is not completely fixed. For instance, the viscous liquid present in the porous skeleton may cause the two media to be loosely bonded. Several authors have attempted to incorporate the effect of imperfect bonding in composite materials. Newmark (1951) modified the perfect interface conditions and explicitly allowed slipping to occur. Similar boundary conditions have been used by Murty (1975) to model the propagation of waves through a loosely-bonded interface by assuming that the interface behaves like a dislocation which preserves the continuity of traction while allowing a finite amount of slip. Jones and Whittier (1967) modelled the wave propagation through a flexibly-bonded interface by allowing both slip and separation. Elastic wave behaviour across linear slip interfaces was discussed by Schoenberg (1980). Martin (1990) studied the linear models of imperfect interfaces between elastic bodies for inclusions and laminated structure and a brief review of imperfect plane interfaces was given therein. Olsson (1990) and Olsson *et al.* (1990) studied some elastodynamic scattering problems.

In this paper, we shall be investigating the problem of reflection and refraction of plane waves incident at a loosely-bonded interface between an elastic solid and a liquid-saturated porous solid half-space. The amplitude and energy ratios are computed numerically from the experimental data given by Fatt (1959) and Yew and Jogi (1976) for a kerosene-saturated sandstone. The dissipation of energy for different degrees of bonding has been shown. This investigation shows that energy is dissipated along the imperfect interfaces while crossing from a porous medium to a simply elastic medium.

The existence of such a medium in the earth cannot be ruled out and so it is reasonable to assume the boundary is loosely bonded. The results of this study may give better information about the porous layers saturated with oil and water in the crust of the earth. It may be mentioned that the application of elastic waves for the detection and study of flaws in materials is of considerable importance in non-destructive evaluations (Griffith, 1920).

BASIC ASSUMPTIONS

Murty (1975, 1976) defined a real parameter (bonding parameter) to which numerical values can be assigned corresponding to a given degree of bonding between half-spaces and discussed the particular cases of an ideally smooth and fully bonded interfaces corresponding to the values 0 and ∞ of this bonding constant. The study is carried out under the following basic assumptions:

(1) the half-spaces are homogeneous and isotropic;
(2) the traction is continuous across the interface;
(3) a finite amount of slip can take place at the interface when periodic waves are propagating;
(4) the slip at the interface is proportional to the local shearing stress. This assumption implies that different degrees of looseness of the interface correspond to different values of the constant of proportionality. This can be written as

$$\text{Shearing stress at the interface} = K \times \text{slip}, \tag{1}$$

where K is a suitable proportionality factor so that the vanishing of K corresponds to an ideally smooth interface, and when K tends to infinity, the stress and strain remain finite and the slip will tend to zero (bonded interface). The intermediate values of K represent a loosely-bonded interface.

We assume the model to consist of a thin viscous liquid layer between an elastic solid half-space and liquid-saturated porous solid half-space. Let T be the thickness of the layer and ξ be the coefficient of viscosity and $T \to 0$ implies that the thickness of the liquid layer is infinitesimally small. It can be assumed that the shearing stress at the interface is given by

$$\tau_{xz} = \xi\left(\frac{\partial \dot{u}}{\partial z}\right), \tag{2}$$

where $\dot{u}$ is the component of velocity parallel to the interface (the dot represents the time derivative) and the partial derivative is taken along the normal to the interface. Equation (2) can be written approximately as

$$\tau_{xz} = \left(\frac{\xi}{T}\right)(\dot{u} - \dot{u}_e), \tag{2a}$$

where $(\dot{u} - \dot{u}_e)$ is the jump in the x-component of velocity across the layer. Assuming the wave motion to be time harmonic, eqn (2a) can be written as

$$\tau_{xz} = i\omega\left(\frac{\xi}{T}\right)(u - u_e), \tag{3}$$

where ω is the angular frequency and the difference $(u - u_e)$ of the displacement components parallel to the interface represents the "slip".

BASIC EQUATIONS

Biot's equations (1956) governing the displacements $\mathbf{u}$ of the skeleton and $\mathbf{U}$ of the interstitial liquid, which together form the saturated porous medium, may be written as

$$N\nabla^2\mathbf{u}+\operatorname{grad}[(D+N)e+Q\varepsilon] = \frac{\partial^2}{\partial t^2}[\rho_{11}\mathbf{u}+\rho_{12}\mathbf{U}]+b\frac{\partial}{\partial t}(\mathbf{u}-\mathbf{U}),$$

$$\operatorname{grad}[Qe+R\varepsilon] = \frac{\partial^2}{\partial t^2}[\rho_{12}\mathbf{u}+\rho_{22}\mathbf{U}]-b\frac{\partial}{\partial t}(\mathbf{u}-\mathbf{U}), \tag{4}$$

where $e = \operatorname{div}\mathbf{u}$ and $\varepsilon = \operatorname{div}\mathbf{U}$.

D, N, Q and R are the elastic constants for the solid–fluid aggregate; ρ_{11}, ρ_{12} and ρ_{22} are dynamical coefficients related to density of solid matrix, density of interstitial liquid and porosity p; b is a dissipation function and is related to Darcy's coefficient of permeability γ by

$$b = \frac{\xi p^2}{\gamma}, \tag{5}$$

where ξ is the fluid viscosity.

The constitutive equations for a liquid-saturated porous solid (Biot, 1956a) are given by

$$\tau_{ij} = (De+Q\varepsilon)\delta_{ij}+2Ne_{ij}, \quad (i,j = 1,2,3)$$

and

$$\tau = Qe+R\varepsilon, \tag{6}$$

where τ_{ij} are the stresses in the solid and τ that in liquid, and

$$e_{ij} = \tfrac{1}{2}(u_{i,j}+u_{j,i}), \tag{7}$$

where e and ε are the dilatation in the solid and liquid, respectively.

Following the assumptions for the porous material (Biot, 1956a) that Poiseuille flow breaks down for frequencies higher than

$$f_t = \frac{\pi v}{4d'^2}, \tag{8}$$

where d' is the diameter of the pore and $v = \xi/\rho_f$ is the kinematic viscosity. We introduce a characteristic frequency

$$f_c = \frac{b}{2\pi p\rho_f},$$

where b is given by $(32\xi p/d'^2)$ with the assumption that pores behave like circular tubes with diameter d'.

In this case, we have

$$\frac{f_t}{f_c} = \frac{\pi^2}{64} = 0.154.$$

For the low frequency range, i.e. for the frequency $\eta < f_t$, we have

$$\frac{\eta}{f_c} < \frac{f_t}{f_c} = 0.154.$$

For a homogeneous isotropic elastic solid (Bullen, 1963), the equation of motion can be written as

$$(\lambda+2\mu)\,\text{grad div}\,\mathbf{u}_e-\mu\,\text{curl curl}\,\mathbf{u}_e=\rho_e\frac{\partial^2\mathbf{u}_e}{\partial t^2}, \tag{9}$$

where $\mathbf{u}_e$ is the displacement vector; λ, μ are the Lamé's constant and ρ_e is the density of the solid.

Stresses $(\sigma_e)_{ij}$ in the elastic solid are related to the strain ε_{ij} by the relation

$$(\sigma_e)_{ij}=\lambda\varepsilon_{kk}\delta_{ij}+2\mu\varepsilon_{ij}, \tag{10}$$

where

$$\varepsilon_{ij}=\tfrac{1}{2}[(u_e)_{i,j}+(u_e)_{j,i}],\quad (i,j=1,2,3).$$

We now consider a Helmholtz resolution of each of the two displacement vectors as:

$$\mathbf{u}=\text{grad}\,\phi+\text{curl}\,\mathbf{H},$$
$$\mathbf{U}=\text{grad}\,\Psi+\text{curl}\,\mathbf{G}.$$

Substituting these displacements in eqns (4) and following Sharma and Gogna (1992), we obtain

$$(\nabla^2+\delta_j^2)\phi_j=0,\quad (j=1,2)$$
$$(\nabla^2+\delta_3^2)\mathbf{H}=0, \tag{11}$$

where

$$\delta_j^2=\frac{\rho\omega^2}{H}\Lambda_j,\quad (j=1,2,3)$$

and for $\Delta^2=B^2-4AC+2if(B-2A)-f^2$,

$$\Lambda_1=\frac{B+if-\Delta}{2A},$$
$$\Lambda_2=\frac{B+if+\Delta}{2A},$$
$$\Lambda_3=\frac{H}{N}\left(\frac{C+if}{\gamma_{22}+if}\right). \tag{12}$$

The other potentials can be written as

$$\Psi=\mu_1\phi_1+\mu_2\phi_2\quad\text{and}\quad \mathbf{G}=\mu_3\mathbf{H},$$

where

$$\mu_j=\frac{g+idf-A\Lambda_j}{h+idf},\quad (j=1,2)$$

and

$$\mu_3=-\left(\frac{\gamma_{12}-if}{\gamma_{22}+if}\right). \tag{13}$$

It is evident from eqns (11) that in a liquid-saturated porous medium, we have two dilatational waves along with one shear wave with phase velocities given by

$$c_j=\left(\frac{H}{\rho}\right)^{1/2}\Big/\text{Re}\,(\Lambda_j^{1/2}),\quad (j=1,2,3). \tag{14}$$

The quantities with the subscript $j = 1$ correspond to the P_{I}, dilatational wave of the first kind (Biot, 1956a), those with $j = 2$, to the P_{II}, dilatational wave of second kind and the quantities with subscript $j = 3$ correspond to the shear waves.

For a homogeneous isotropic elastic solid, considering the Helmholtz resolution for the displacement

$$\mathbf{u}_e = \operatorname{grad} \phi_e + \operatorname{curl} \mathbf{\Psi}_e, \tag{15}$$

the potential functions are found to satisfy the wave equations

$$\nabla^2 \phi_e = \frac{1}{\alpha^2} \frac{\partial^2 \phi_e}{\partial t^2}, \quad \text{and} \quad \nabla^2 \mathbf{\Psi}_e = \frac{1}{\beta^2} \frac{\partial^2 \mathbf{\Psi}_e}{\partial t^2}, \tag{16}$$

where

$$\alpha = \sqrt{\frac{\lambda + 2\mu}{\rho_e}} \quad \text{and} \quad \beta = \sqrt{\frac{\mu}{\rho_e}}.$$

FORMULATION OF THE PROBLEM

We consider a model consisting of an isotropic homogeneous impervious elastic solid half-space and a liquid-saturated porous solid half-space separating at a loosely bonded plane interface $z = 0$ with the z-axis pointing into the liquid-saturated porous solid as shown in Fig. 1(a).

We shall consider here the case when the incident wave propagates through the liquid-saturated porous solid (Medium I). Incident P_{I} or P_{II} or SV wave in medium I will give reflected P_{I}, P_{II} and SV waves making complex angles θ_1^*, θ_2^* and θ_3^* with the normal to the interface respectively and also transmitted P and SV waves in medium II at angles θ_1 and θ_2 respectively, as depicted in Fig. 1(a).

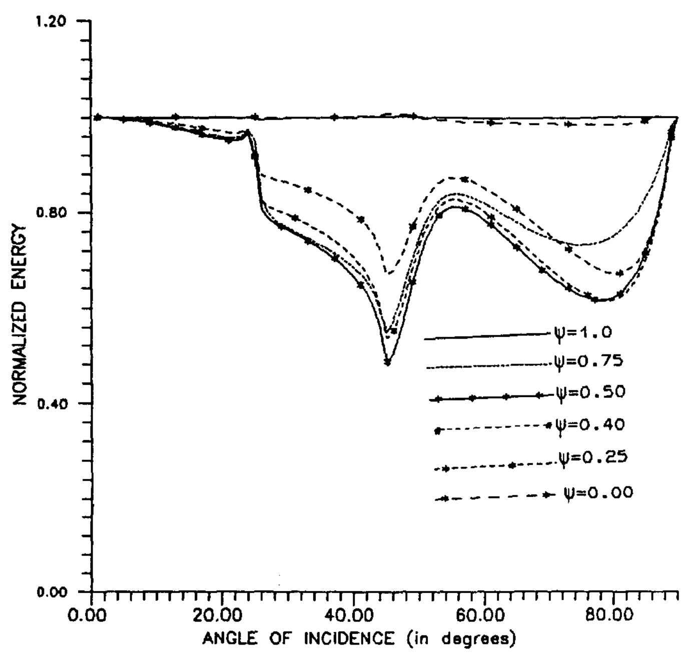

Fig. 1. Variation of normalized energy with incidence angle of P_{I} wave for different values of the bonding constant.

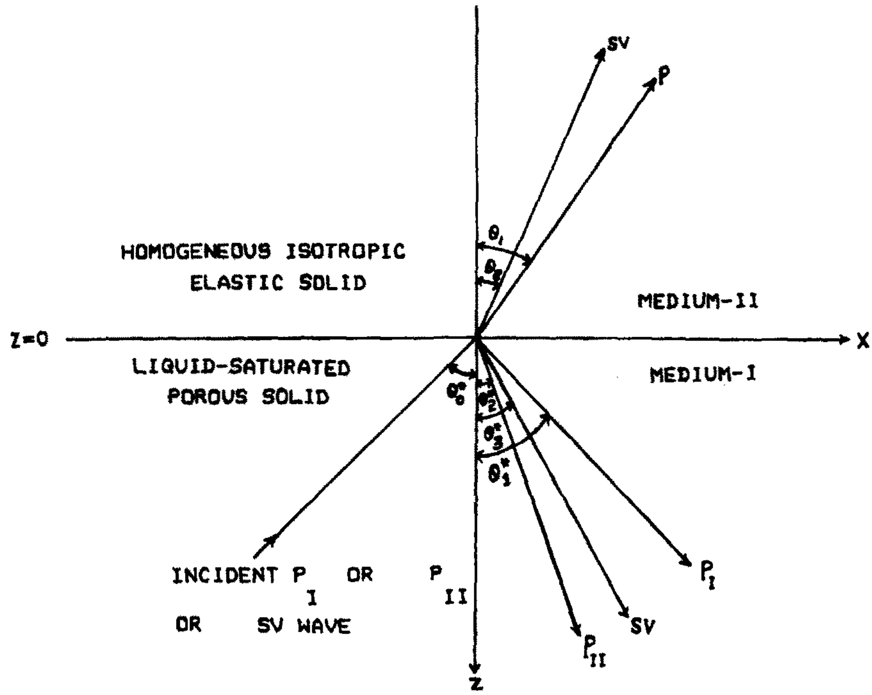

Fig. 1(a). Geometry of the problem.

The displacement potentials, in medium I, satisfying eqns (11) can be written as

$$\phi_1 = A_0 \exp[i\{\delta_1(x \sin\theta_0^* - z\cos\theta_0^*) - \omega t\}] + A_1 \exp[i\{\delta_1(x\sin\theta_1^* + z\cos\theta_1^*) - \omega t\}],$$
$$\phi_2 = B_0 \exp[i\{\delta_2(x \sin\theta_0^* - z\cos\theta_0^*) - \omega t\}] + B_1 \exp[i\{\delta_2(x\sin\theta_2^* + z\cos\theta_2^*) - \omega t\}],$$
$$\phi_3 = C_0 \exp[i\{\delta_3(x \sin\theta_0^* - z\cos\theta_0^*) - \omega t\}] + C_1 \exp[i\{\delta_3(x\sin\theta_3^* + z\cos\theta_3^*) - \omega t\}], \tag{17}$$

where $\phi_3 = (-\mathbf{H})_y$.

A_0 (or B_0 or C_0), A_1, B_1 and C_1 are the amplitudes of the incident P_I (or P_{II} or SV) wave, reflected P_I, P_{II} and SV waves, respectively.

The displacements $\mathbf{u} = (u, 0, w)$ and $\mathbf{U} = (U, 0, W)$ can be written in terms of potentials as

$$u = \frac{\partial\phi_1}{\partial x} + \frac{\partial\phi_2}{\partial x} + \frac{\partial\phi_3}{\partial z},$$
$$w = \frac{\partial\phi_1}{\partial z} + \frac{\partial\phi_2}{\partial z} - \frac{\partial\phi_3}{\partial x},$$
$$U = \mu_1\frac{\partial\phi_1}{\partial x} + \mu_2\frac{\partial\phi_2}{\partial x} + \mu_3\frac{\partial\phi_3}{\partial z},$$
$$W = \mu_1\frac{\partial\phi_1}{\partial z} + \mu_2\frac{\partial\phi_2}{\partial z} - \mu_3\frac{\partial\phi_3}{\partial x}. \tag{18}$$

In medium II, the displacement potentials satisfying eqn (16) are given by

$$\phi_e = A_e \exp\left[i\omega\left(\frac{x\sin\theta_1 - z\cos\theta_1}{\alpha} - t\right)\right],$$
$$\Psi_e = B_e \exp\left[i\omega\left(\frac{x\sin\theta_2 - z\cos\theta_2}{\beta} - t\right)\right]. \tag{19}$$

The displacement components in medium II are given as

$$u_e = \frac{\partial \phi_e}{\partial x} + \frac{\partial \Psi_e}{\partial z}, \quad w_e = \frac{\partial \phi_e}{\partial z} - \frac{\partial \Psi_e}{\partial x}. \tag{20}$$

BOUNDARY CONDITIONS

We shall now discuss the boundary conditions along the loosely bonded interface $z = 0$ between a semi-infinite impervious elastic solid and a liquid-saturated porous half-space. Following Deresiewicz and Skalak (1963) and Murty (1975), the boundary conditions are:

(1) continuity of the stresses and normal displacement across the interface $z = 0$;

(2) vanishing of the normal velocity of the liquid relative to the solid in the porous aggregate. This assures us that liquid does not flow into the elastic solid, which is assumed to be impervious, along the interface;

(3) shearing stress is proportional to the slip at the interface.

The boundary condition (3) can be written as

$$\tau_{xz} = -ikN\left(\frac{\zeta}{\sin\theta_0}\right)(u-u_e), \tag{21}$$

where

$$\zeta = \frac{V_0\xi}{NT}, \quad \text{and} \quad \theta_0 = \sin^{-1}\left(\frac{V_0 k}{\omega}\right)$$

is the angle of incidence, and V_0, the phase velocity of the incident wave, takes the values c_1, c_2, c_3 for incident P_{I}, P_{II} and SV waves, respectively.

It is convenient to introduce a variable Ψ^*, $0 \leqslant \Psi^* \leqslant 1$, such that

$$\zeta = \frac{\Psi^*}{(1-\Psi^*)}. \tag{22}$$

The range $0 \leqslant \zeta \leqslant \infty$ shall be mapped on the range $0 \leqslant \Psi^* \leqslant 1$. Thus $\Psi^* = 0$ corresponds to a smooth interface and $\Psi^* = 1$ corresponds to a welded interface between half-spaces. Ψ^* may be considered as a bonding constant.

The boundary conditions at the interface $z = 0$ can be written as

$$\begin{aligned} &(1)\ \tau_{zz}+\tau = (\sigma_e)_{zz}, \qquad &&(2)\ \tau_{xz} = (\sigma_e)_{xz}, \\ &(3)\ w = w_e, &&(4)\ \dot{w}-\dot{W} = 0, \\ &(5)\ \tau_{xz} = -ikN\frac{\Psi^*}{(1-\Psi^*)}\frac{1}{\sin\theta_0}(u-u_e). \end{aligned} \tag{23}$$

Following Schoenberg (1971), Snell's law may be written as

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1^*}{V_1^*} = \frac{\sin\theta_2^*}{V_2^*} = \frac{\sin\theta_3^*}{V_3^*} = \frac{\sin\theta_1}{\alpha} = \frac{\sin\theta_2}{\beta}, \tag{24}$$

where

$$V_j^* = \frac{\omega}{\delta_j}, \quad (j = 1,2,3).$$

Making use of eqns (17)–(20), the boundary conditions (23) give rise to a set of five non-homogeneous equations, which on solving give the amplitude ratios of reflected and transmitted waves. A computer program was developed for carrying out the calculations.

Following Achenbach (1973), the energy ratios E_i, $(i = 1,2,\ldots,5)$, for the reflected P_{I}, P_{II}, SV; refracted P and SV waves respectively, are obtained as:

$$E_i = \langle P_i^* \rangle / \langle P_0^* \rangle, \quad (i = 1,2,\ldots,5), \tag{25}$$

where for

$$1/V_j^{*2} = q_{jR}+iq_{jI}, \quad \cos\theta_j^*/V_j^* = c_{jR}+ic_{jI} \tag{26}$$

and

$$\mu_j = \mu_{jR}+i\mu_{jI}, \quad (j = 1,2,3)$$

we have

$$\langle P_1^* \rangle = \tfrac{1}{2}\omega^4[(P+2Q\mu_{1R}+R\mu_{1R}^2+R\mu_{1I}^2)(q_{1R}c_{1R}+q_{1I}c_{1I})]|Z_1|^2,$$

$$\langle P_2^* \rangle = \tfrac{1}{2}\omega^4[(P+2Q\mu_{2R}+R\mu_{2R}^2+R\mu_{2I}^2)(q_{2R}c_{2R}+q_{2I}c_{2I})]|Z_2|^2,$$

$$\langle P_3^* \rangle = \tfrac{1}{2}\omega^4 N(q_{3R}c_{3R}+q_{3I}c_{3I})|Z_3|^2,$$

$$\langle P_4^* \rangle = \tfrac{1}{2}\omega^4\rho_e\frac{1}{V_0}\sqrt{(V_0/\alpha)^2-\sin^2\theta_0}|Z_4|^2,$$

$$\langle P_5^* \rangle = \tfrac{1}{2}\omega^4\rho_e\frac{1}{V_0}\sqrt{(V_0/\beta)^2-\sin^2\theta_0}|Z_5|^2,$$

$$\langle P_0^* \rangle = \tfrac{1}{2}\omega^4[(P+2Q\mu_{jR}+R\mu_{jR}^2+R\mu_{jI}^2)(q_{jR}c_{jR}+q_{jI}c_{jI})],$$

in which

$$j = \begin{cases} 1 \text{ for incident } P_{\text{I}} \text{ wave} \\ 2 \text{ for incident } P_{\text{II}} \text{ wave} \end{cases}$$

and

$$\langle P_0^* \rangle = \tfrac{1}{2}\omega^4 N(q_{3R}c_{3R}+q_{3I}c_{3I}) \text{ for incident } SV \text{ wave.} \tag{27}$$

$|Z_i|$, $(i = 1,2,\ldots,5)$, are the amplitude ratios of reflected P_{I}, P_{II}, SV; refracted P and SV waves to that of incident wave, respectively.

NUMERICAL RESULTS AND DISCUSSION

For computing the amplitude and energy ratios, we consider the model consisting of kerosene-saturated sandstone loosely bonded with granite which is assumed to be an impervious elastic solid.

Following the experimental results given by Yew and Jogi (1976) for a kerosene-saturated sandstone, we take the following values of the relevant parameters:

$$D = 0.445\times10^{10}\ \text{N m}^{-2},$$

$$Q = 0.0744\times10^{10}\ \text{N m}^{-2},$$

$$R = 0.0326\times10^{10}\ \text{N m}^{-2},$$

$$N = 0.2765\times10^{10}\ \text{N m}^{-2},$$

$$\rho_s = 2.6\times10^3\ \text{kg m}^{-3},$$

$$\rho_f = 0.82\times10^3\ \text{kg m}^{-3}$$

and

$$\rho_{12} = -0.0001\rho,$$

where $\rho = 2.1372\times10^3$ kg m^{-3} is the mass density of the aggregate.

The elastic parameters for granite (Bullen, 1963) are given by

$$\lambda = 2.238\times10^{10}\ \text{N m}^{-2},$$

$$\mu = 2.992\times10^{10}\ \text{N m}^{-2},$$

$$\rho_e = 2.65\times10^3\ \text{kg m}^{-3}.$$

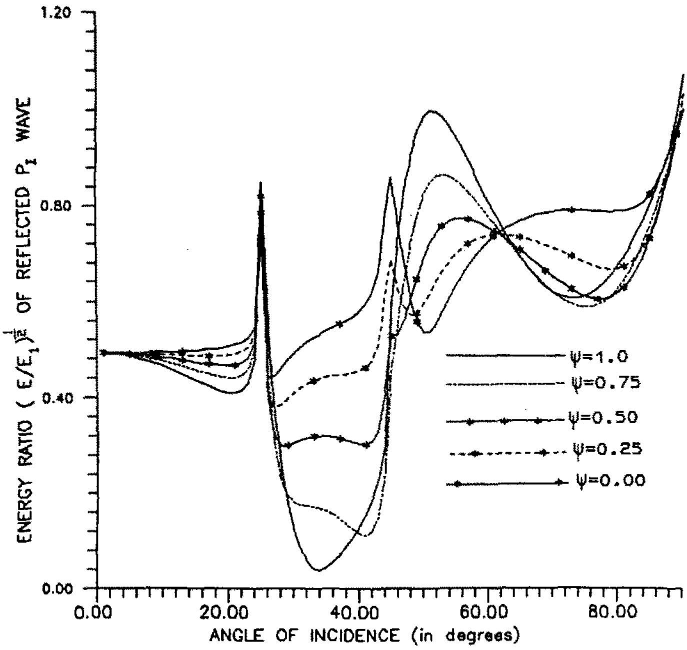

Fig. 2. Variation of energy ratios with incidence angle of P_I wave for different values of the bonding constant.

Corresponding to the above given values of parameters, the energy ratios of different reflected and refracted waves for different values of bonding constant are computed using the relations (25)–(27). The results have been discussed for the following two cases.

Case (i) : Incident P_I *wave*

Variation of normalized energy (sum of energy ratios of different reflected and transmitted waves to that of incident P_I wave) with the angle of incidence for different values of Ψ^* ranging from 0 to 1 is shown in Fig. 1. The energy dissipation is negligible for $\Psi^* = 0.0$ and $\Psi^* = 1.0$ as expected in the case of smooth interface and welded contact. For the loosely bonded interface ($\Psi^* = 0.75, 0.50, 0.40, 0.25$), the energy is dissipated. The dissipation is

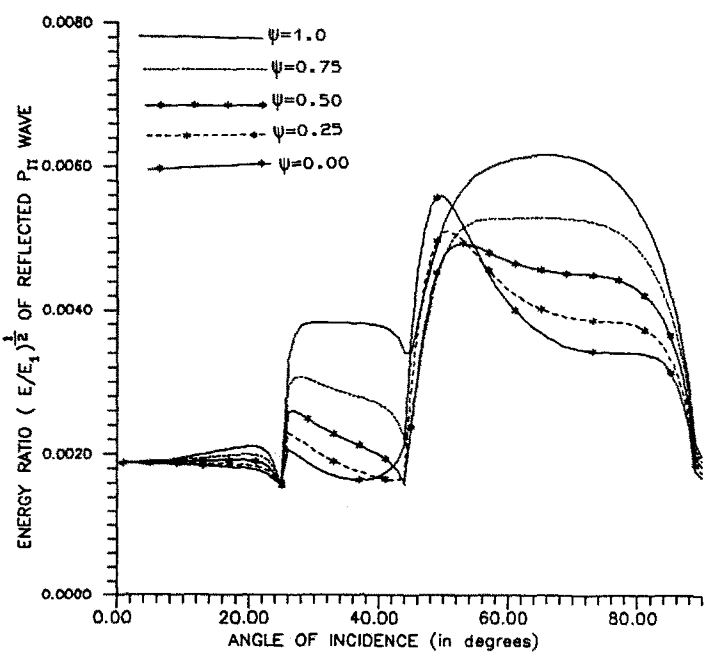

Fig. 3. Variation of energy ratios with incidence angle of P_I wave for different values of the bonding constant.

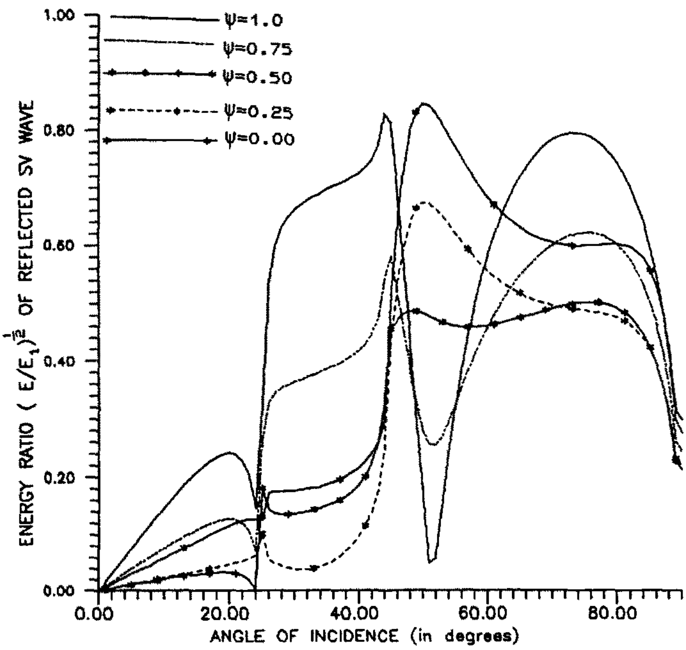

Fig. 4. Variation of energy ratios with incidence angle of P_I wave for different values of the bonding constant.

maximum when $\Psi^* = 0.5$ and it decreases when we move either from 0.5 to 1.0 or from 0.5 to 0.0, i.e. when the loosely bonded interface approaches an ideal interface. Thus one can say that a loosely bonded interface acts as an absorber of energy while a smooth interface and a fully bonded interface absorb no energy.

The nature of dependence of energy ratios of reflected and transmitted waves is shown in Figs 2–6. It is noted that the energy ratio of each of the reflected and refracted waves is different for different values of Ψ^* except at normal and grazing incidence. It is clear from

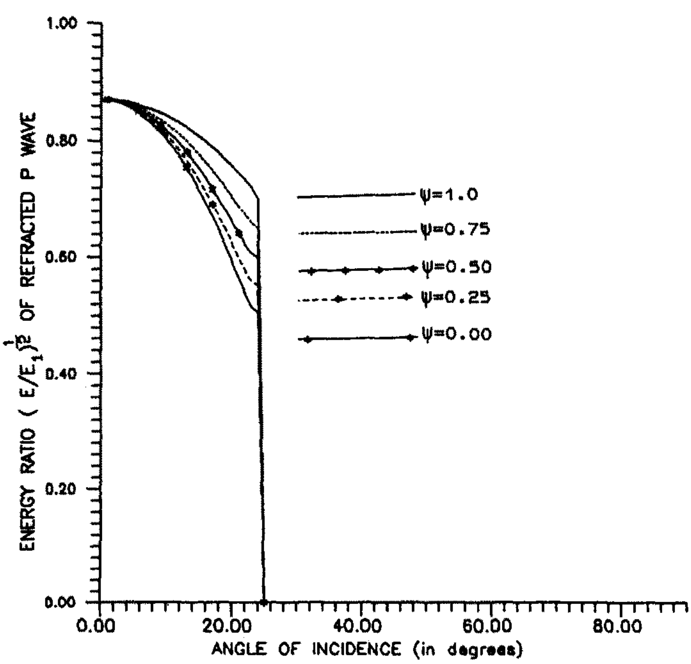

Fig. 5. Variation of energy ratios with incidence angle of P_I wave for different values of the bonding constant.

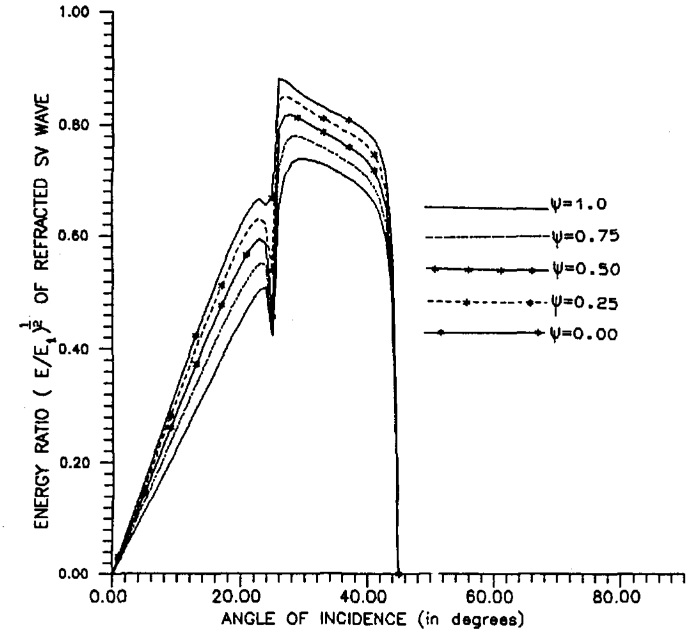

Fig. 6. Variation of energy ratios with incidence angle of P_{I} wave for different values of the bonding constant.

Fig. 5 that the critical angle of the incident P_{I} wave for the refracted P wave is nearly 25°. The critical angle for a refracted SV wave is found to be nearly 45° (Fig. 6).

Case (ii) : Incident SV wave

The dependence of normalized energy on the angle of incidence of the incident SV wave for different degrees of looseness ($\Psi^* = 0.0, 0.25, 0.50, 0.75, 1.0$) has been shown in Fig. 7. The energy is dissipated as we move away from the condition of ideal interface

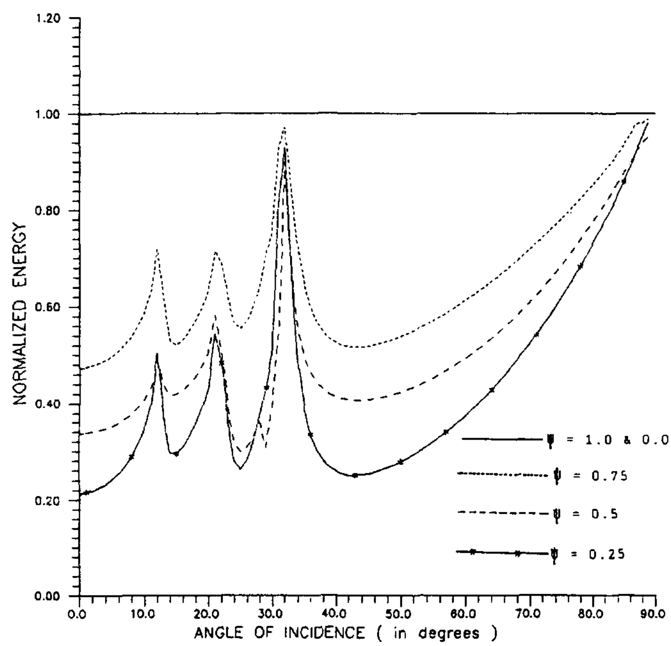

Fig. 7. Variation of normalized energy with incidence angle of SV wave for different values of the bonding constant.

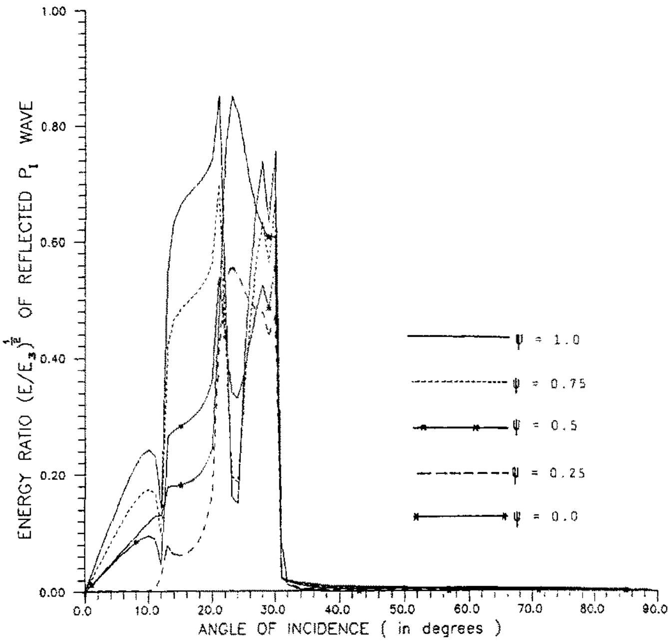

Fig. 8. Variation of energy ratios with incidence angle of SV wave for different values of the bonding constant.

(smooth or welded) to the loosely bonded interface. It is evident from the figure that there is a sharp increase in dissipation at 10°, 20° and 30°, angle of incidence for $\Psi^* = 0.25, 0.50$ and 0.75. It may be noted that these are the critical angles for refracted P, refracted SV and reflected P_I waves, respectively.

The energy ratios of different reflected and refracted waves for different values of the bonding parameter are shown in Figs 8–12. As the velocities of reflected P_I, refracted P and refracted SV waves are greater than the velocity of the incident SV wave, three critical angles of incident SV wave exist. These angles are approximately 30°, 10° and 20°,

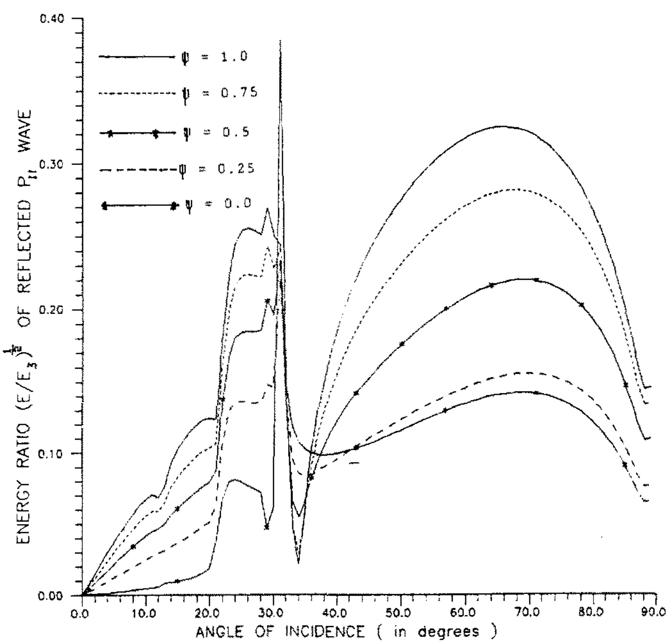

Fig. 9. Variation of energy ratios with incidence angle of SV wave for different values of the bonding constant.

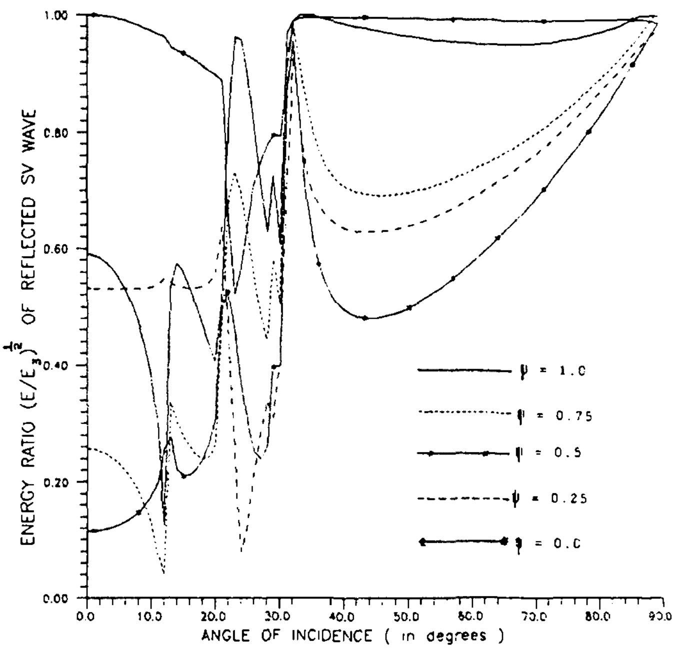

Fig. 10. Variation of energy ratios with incidence angle of SV wave for different values of the bonding constant.

respectively. The critical angle for a reflected P_1 wave in the porous medium is not as sharp as in the case of the elastic medium. The energy transmitted to the P_1 wave is not zero beyond the critical angle (though it is very small).

CONCLUSIONS

In this paper, the theoretical problem of the reflection and refraction of plane waves in a medium consisting of a liquid-saturated porous solid loosely bonded with an elastic

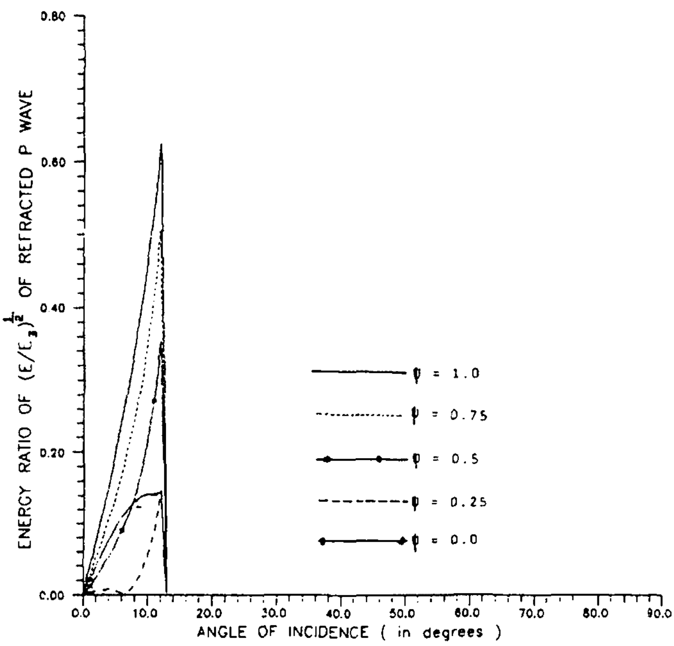

Fig. 11. Variation of energy ratios with incidence angle of SV wave for different values of the bonding constant.

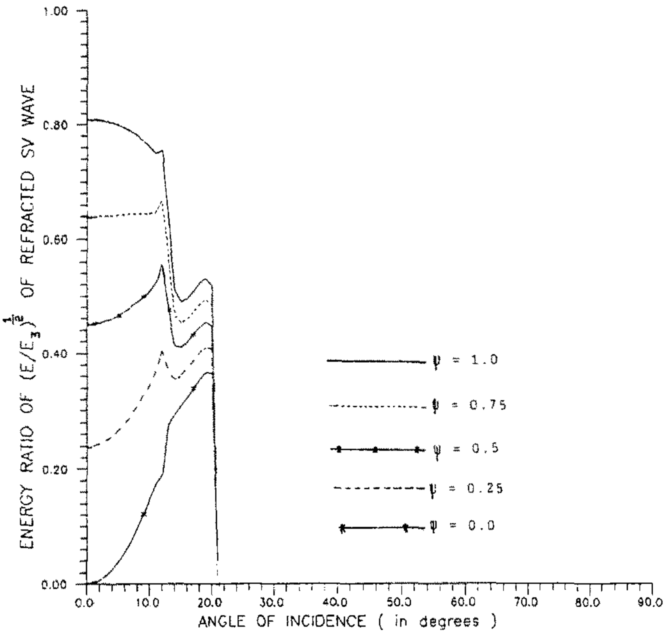

Fig. 12. Variation of energy ratios with incidence angle of SV wave for different values of the bonding constant.

solid has been studied and detailed numerical calculations have been performed for the case of incident P_{I}, P_{II} and SV waves for different degrees of looseness of contact between the two half-spaces considered. The cases of smooth interface and welded contact between the two half-spaces have been obtained as particular cases. It has been observed that a loosely bonded interface acts as an absorber of energy. The amplitude ratios and energy ratios of different reflected and refracted waves change with the bonding parameter Ψ^*. It can be concluded that the deviation from the assumption of welded interface to loosely bonded interface affects the reflection-transmission phenomenon.

Acknowledgements—The authors are thankful for the facilities provided by the Department of Science and Technology, New Delhi under Project No. DST/23(19)/ESS/90. Earlier, the first author was working as a Research Associate in the project.

REFERENCES

Achenbach, J. D. (1973). *Wave Propagation in Elastic Solids*. North-Holland, Amsterdam.
Biot, M. A. (1956a). General solutions of the equations of elasticity and consolidation for a porous material. *J. Appl. Mech.* **23**, 91–95.
Biot, M. A. (1956b). The theory of propagation of elastic waves in fluid-saturated porous solids. *J. Acoust. Soc. Am.* **28**, 168–191.
Biot, M. A. (1962a). Mechanics of deformation and acoustic propagation in porous media. *J. Appl. Phys.* **33**, 1482–1498.
Biot, M. A. (1962b). Generalized theory of acoustic propagation in porous dissipative media. *J. Acoust. Soc. Am.* **34**, 1256–1264.
Bullen, K. E. (1963). *An Introduction to the Theory of Seismology*. Cambridge University Press, Cambridge.
Deresiewicz, H. (1960). The effect of boundaries on wave propagation in a liquid-filled porous solid—I. *Bull. Seism. Soc. Am.* **50**, 599–607.
Deresiewicz, H. and Levy, A. (1967). The effect of boundaries on wave propagation in a liquid-filled porous solid—X: Transmission through a stratified medium. *Bull. Seism. Soc. Am.* **57**, 381–392.
Deresiewicz, H. and Rice, J. T. (1962). The effect of boundaries on wave propagation in a liquid-filled porous solid—III: Reflection of plane waves at a free plane boundary. *Bull. Seism. Soc. Am.* **52**, 595–626.
Deresiewicz, H. and Rice, J. T. (1964). The effect of boundaries on wave propagation in a liquid-filled porous solid—V: Transmission across a plane interface. *Bull. Seism. Soc. Am.* **54**, 409–416.
Deresiewicz, H. and Skalak, R. (1963). On uniqueness in dynamic poroelasticity. *Bull. Seism. Soc. Am.* **53**, 783–789.
Fatt, I. (1959). Biot–Willis elastic coefficients for a sandstone. *J. Appl. Mech.* **26**, 296–297.
Geertsma, J. and Smit, D. C. (1961). Some aspects of elastic wave propagation in a fluid-saturated porous solid. *Geophys.* **26**, 169–181.
Griffith, A. A. (1920). Phenomena of rupture and flow of solids. *Phil. Trans. Roy. Soc.* **A221**, 163.

Hajra, S. and Mukhopadhyay, A. (1982). Reflection and refraction of seismic waves incident obliquely at the boundary of a liquid-saturated porous solid. *Bull. Seism. Soc. Am.* **72,** 1509–1533.

Jones, J. P. and Whittier, J. S. (1967). Waves at a flexibly bonded interface. *J. Appl. Mech.* **34,** 905–909.

Martin, P. A. (1990). Thin interface layer: Adhesives, approximations and analysis. In *Elastic Waves and Ultrasonic Evaluation* (Edited by S. K. Datta, J. D. Achenbach and Y. S. Rajapakse). North-Holland, Amsterdam.

Murty, G. S. (1975). A theoretical model for the attenuation and dispersion of Stoneley waves at the loosely bonded interface of elastic half-spaces. *Phys. Earth Planet. Intl* **11,** 65–79.

Murty, G. S. (1976). Reflection, transmission and attenuation of elastic waves at a loosely bonded interface of two half-spaces. *Geophys. J. Roy. Astro. Soc.* **44,** 389–404.

Newmark, N. M., Siess, C. P. and Viest, I. M. (1951). Tests and analysis of composite beams with incomplete interaction. *Proc. Soc. Expl Stress Anal.* **9**(1), 75–92.

Olsson, P. (1990). Elastodynamic scattering from penny-shaped and nonplanar cracklike flaws with dissipation and restoring forces. *J. Appl. Mech.* **57,** 661–666.

Olsson, P., Datta, S. K. and Bostoom, A. (1990). Elasto-dynamic scattering from inclusions surrounded by thin interface layers. *J. Appl. Mech.* **57,** 672–676.

Persson, G. and Olsson, P. (1991). 2-D elastodynamic scattering from a semi-infinite cracklike flaw with interfacial forces. *Wave Motion* **13,** 21–42.

Schoenberg, M. (1971). Transmission and reflection of plane wave at an elastic-viscoelastic interface. *Geophys. J. Roy. Astr. Soc.* **25,** 35–47.

Schoenberg, M. (1980). Elastic wave behaviour across linear slip interfaces. *J. Acoust. Soc. Am.* **68,** 1516–1521.

Sharma, M. D. and Gogna, M. L. (1992). Reflection and refraction of plane harmonic waves at an interface between elastic solid and porous solid saturated by viscous liquid. *PAGEOPH* **138**(2), 249–266.

Yew, C. H. and Jogi, P. N. (1976). Study of wave motions in fluid-saturated porous rocks. *J. Acoust. Soc. Am.* **60,** 2–8.